Pure math – Model 2

1. The coefficient of T_5 in the expansion of $(1 + 2x)^{10}$ according to the ascending power of x is ...

a) $16 \times {}^{10}C_5$

b) $\frac{1}{16} \times {}^{10}C_5$

c) $16 \times {}^{10}C_4$

d) $\frac{1}{16} \times {}^{10}C_4$

2. The distance between the point $(6,7,8)$ and the y-axis is ...

a) 12 b) 10 c) 8 d) 6

3. If $\sin x = \cos y$, where $x, y \in]0, \pi[$, then $\frac{dy}{dx} = \cdots$

a) zero b) -1 c) $\frac{\pi}{2}$ d) $\frac{-\cos x}{\sin y}$

4. $\int e^{\sec^2 x - \tan^2 x} dx = \cdots$

a) zero b) e^x c) ex d) e

5. The value of the term free of x in the expansion $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ equals ...

a) 210 b) 105 c) 70 d) 112

6. If $\vec{A} = (-2,0,3), \vec{B} = (4,2,-5)$, then $\overrightarrow{AB} = \cdots$

a) $(-6,-2,8)$

b) $(2,2,-2)$

c) $(6,2,-8)$

d) $(1,1,-1)$

7. If $y = x\sin x$, then $x\frac{d^3y}{dx^3} + x\frac{dy}{dx} = \cdots$

a) $2x$ b) $2y$ c) $3xy$ d) $-2y$

8. The volume of the solid generated by rotating the region bounded between $y = x^3 + 1, y = 0, x = 1$ a complete revolution about the x-axis $= \cdots$ cubic units

a) $\frac{14}{23}\pi$
b) $\frac{16}{7}\pi$
c) $\frac{20}{23}\pi$
d) $\frac{11}{23}\pi$

9. The value of $\log_{16}\left(\frac{4+\omega+2\omega^2}{\omega^2+1} + \frac{\omega^2-1}{2+\omega+2\omega^2}\right) = \cdots$

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) $\frac{1}{3}$
d) 1

10. If $\| \vec{A} \times \vec{B} \|^2 + (\vec{A} \cdot \vec{B})^2 = 144$ and $\| \vec{A} \| = 4$, then $\| \vec{B} \| = \cdots$
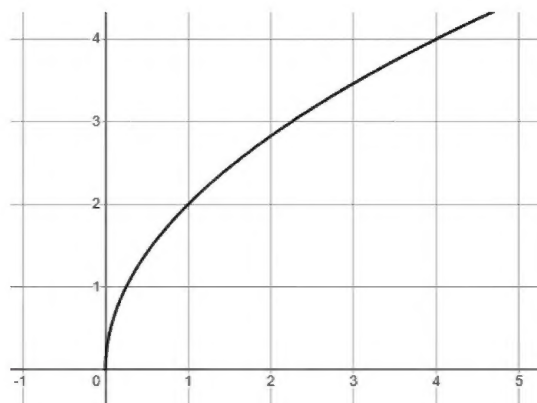
e) 3
b) 4
c) 5
d) 6

11. A point moves along the curve $x^2 + y^2 - 4x + 8y - 6 = 0$, and the rate of change of the x-coordinate at the point $(3,1)$ is 4, then the rate of change of the y-coordinate is...

a) $\frac{3}{5}$
b) $\frac{4}{5}$
c) $\frac{-4}{5}$
d) $\frac{-3}{5}$

12. The area of the region bounded by the curve $y = \sqrt{kx}$ and the straight lines $x = 9$ and $y = 0$ is .... Square units

a) 6
b) 12
c) 24
d) 36

13. The exponential form of the complex number $z = 2 + 2\sqrt{3}i$ is ...

a) $4e^{-\frac{\pi}{3}i}$
b) $4e^{\frac{\pi}{3}i}$
c) $4e^{-\frac{\pi}{6}i}$
d) $4e^{\frac{\pi}{6}i}$

2

14. $\sin^2 \theta_x + \sin^2 \theta_y + \sin^2 \theta_z = \cdots$
a) -1   b) 1   c) 2   d) 3

15. The curve of the function $f(x) = x^4 - 24x^2 + 4$ is convex downward on the interval ...
a) $]-\infty, 2[$ b) $]-\infty, -2]$
c) $]-2, 2[$ d) $R - [-2, 2]$

16. The trigonometric form of the complex number $z = -\sqrt{3} + i$ is $\cdots$
a) $3(\cos 150° + i\sin 150°)$
b) $2(\cos 150° + i\sin 150°)$
c) $2(\sin 150° + i\cos 150°)$
d) $\cos 150° + i\sin 150$

17. The equation of the plane passing through the point $(1, -2, 5)$ and its normal vector $(2, 1, 3)$ is ...
a) $2x + y + 3z = 1$
b) $2x + y + 3z = 15$
b) $x - 2y + 5z = 15$
d) $x + y + z = 4$

18. The function $f(x) = \dfrac{x^2 + x + 1}{x + 1}$ is decreasing on ...
a) $[-2, 0]$ b) $]-1, \infty[$ c) $]-2, \infty[$ d) $]-2, 0[-\{1\}$

Essay Questions

19. If $a = 2 + 3\omega, b = 2 + 3\omega^2$, then Find the value of ab

20. The sum of three numbers is 36, and the greatest number is twice the smaller, find the three numbers if Their product is maximum.

1 Coeff. of $T_5 = {}^{10}C_4 (2)^4 (1)^{10-4}$

$= {}^{10}C_4 (2)^4 = 16 \, {}^{10}C_4$ ⓒ

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2 distance between the Point $(6,8)$

and the y-axis $= \sqrt{6^2 + 8^2}$

$= 10$ length unit. ⓑ

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3 $\sin x = \cos y$

diff. of both sides w.r.t. x

$\cos x = -\dfrac{dy}{dx} \sin y$

$\therefore \dfrac{dy}{dx} = -\dfrac{\cos x}{\sin y}$. ⓓ

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4 $\displaystyle \int e^{\sec^2 x - \tan^2 x} \, dx$

$= \displaystyle \int e^1 dx = \int e \, dx$

$= e \, x + c.$ ⓒ

$\because 1 + \tan^2 x = \sec^2 x$

$\therefore \sec^2 x - \tan^2 x = 1$

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5 <u>first</u>:

$\dfrac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} = \dfrac{(x^{\frac{1}{3}}+1)(x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1)}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1}$

$= x^{\frac{1}{3}} + 1.$

<u>Second</u>: $\dfrac{x-1}{x - x^{\frac{1}{2}}} = \dfrac{(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}+1)}{x^{\frac{1}{2}}(x^{\frac{1}{2}} - 1)}$

$= \dfrac{x^{\frac{1}{2}}+1}{x^{\frac{1}{2}}}$

$= \dfrac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}} + \dfrac{1}{x^{\frac{1}{2}}} = 1 + x^{-1/2}$

$\therefore$ expansion is $\left(x^{\frac{1}{3}} + 1 - (1 + \frac{1}{x^{1/2}})\right)^{10}$

$= \left(x^{\frac{1}{3}} + 1 - 1 - \frac{1}{x^{1/2}}\right)^{10}$

$= \left(x^{\frac{1}{3}} - x^{-1/2}\right)^{10}$

to find the free term of x

first $\longrightarrow$ find the General term.

$T_{r+1} = {}^{10}C_r \left(-x^{-1/2}\right)^r \left(x^{\frac{1}{3}}\right)^{10-r}$

$= {}^{10}C_r (-1)^r \, x^{-\frac{1}{2}r + \frac{10}{3} - \frac{1}{3}r}$

$\because \dfrac{10}{3} - \dfrac{5}{6} r = 0$

$\dfrac{5}{6} r = \dfrac{10}{3} \Rightarrow \boxed{r = 4}$

$\therefore$ free term is $= T_5$
    of x

$= {}^{10}C_4 (-1)^4$

$= {}^{10}C_4 = 210.$ ⓐ

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6 $\vec{AB} = \vec{B} - \vec{A} = (4, 2, -5) - (-2, 0, 3)$

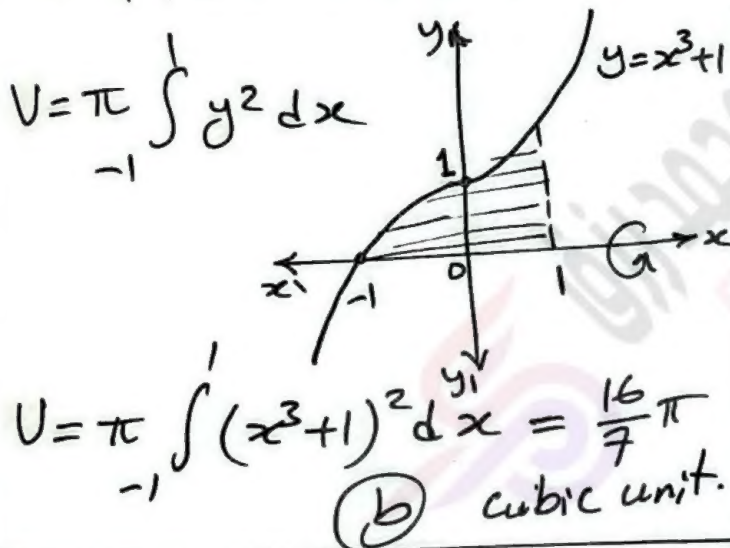$\therefore \vec{AB} = (6, 2, -8).$ ⓒ

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7 $y = x \sin x$

$\dfrac{dy}{dx} = x \cos x + \sin x.$

$\dfrac{d^2 y}{dx^2} = \cos x - x \sin x + \cos x$

$= 2 \cos x - x \sin x.$

$\dfrac{d^3 y}{dx^3} = -2 \sin x - [x \cos x + \sin x]$

$= -2 \sin x - x \cos x - \sin x$

$= -x \cos x - 3 \sin x.$

By Substitution in $l.H.S.$

$\longrightarrow$ (cont.)

follow Question (7):

$$x \frac{d^3y}{dx^3} + x \frac{dy}{dx}$$

$$= x\left[-x \cos x - 3\sin x\right]$$
$$+ x\left[x \cos x + \sin x\right]$$

$$= -x^2 \cos x - 3x \sin x$$
$$+ x^2 \cos x + x \sin x$$

$$= -2x \sin x = -2y. \quad \text{(d)}$$

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8 to find the points of intersection with x-axis ⟹ put $y = 0$

$$x^3 + 1 = 0 \Rightarrow x^3 = -1 \Rightarrow x = -1$$

$$V = \pi \int_{-1}^{1} y^2 \, dx$$

$$y = x^3 + 1$$

$$V = \pi \int_{-1}^{1} (x^3+1)^2 \, dx = \frac{16}{7}\pi$$

$$\text{(b)} \quad \text{cubic unit.}$$

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9 $\log_{16}\left(\dfrac{4+w+2w^2}{w^2+1} + \dfrac{w^2-1}{2+w+2w^2}\right)$
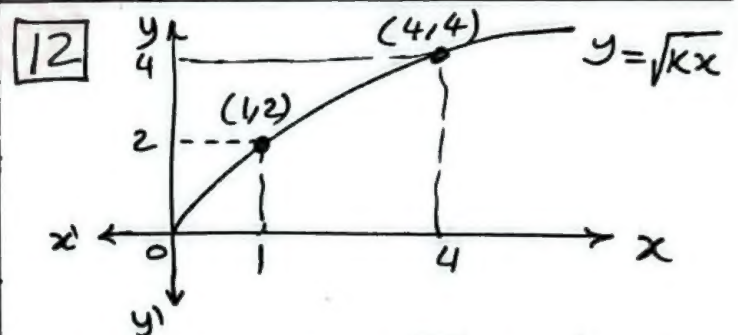
$$= \log_{16}\left(\frac{4+w+2w^2}{-w} + \frac{w^2-1}{+w}\right)$$

$$= \log_{16}\left(\frac{3+w+3w^2}{-w}\right)$$

$$= \log_{16}\left(\frac{w-3w}{-w}\right) = \log_{16}\left(\frac{-2w}{-w}\right)$$

$$= \log_{16} 2 = \frac{1}{4}. \quad \text{(a)}$$

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10 $\|\vec{A}\times\vec{B}\|^2 + (\vec{A}\cdot\vec{B})^2 = 144$

$$\|\vec{A}\|^2\|\vec{B}\|^2 \sin^2\theta + \|\vec{A}\|^2\|\vec{B}\|^2\cos^2\theta = 144$$

$$\|\vec{A}\|^2\|\vec{B}\|^2\left[\sin^2\theta + \cos^2\theta\right] = 144$$

$$\therefore \|\vec{A}\|^2\|\vec{B}\|^2 = 144$$

$$(4)^2 \|\vec{B}\|^2 = 144$$

$$\therefore \|\vec{B}\|^2 = 9 \Rightarrow \therefore \|\vec{B}\| = 3. \quad \text{(a)}$$

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11 at $x=3$, $y=1$, $\dfrac{dx}{dt} = 4$ unit/sec.

$$x^2 + y^2 - 4x + 8y - 6 = 0$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} - 4\frac{dx}{dt} + 8\frac{dy}{dt} = 0$$

$$6(4) + 2\frac{dy}{dt} - 4(4) + 8\frac{dy}{dt} = 0$$

$$10\frac{dy}{dt} + 8 = 0 \quad \text{(c)}$$

$$\frac{dy}{dt} = \frac{-8}{10} = \frac{-4}{5} \text{ units/sec.}$$

12

$$y = \sqrt{kx}$$

at $y = 2 \Rightarrow 2 = \sqrt{k} \Rightarrow k = 4$
and $x = 1$

$$\therefore y = \sqrt{4x} = 2\sqrt{x}$$

$$\text{Area} = \int_0^9 y \, dx = \int_0^9 2\sqrt{x}\, dx$$

$$= 36 \text{ Square unit.} \quad \text{(d)}$$

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13 $Z = 2 + 2\sqrt{3}\, i \longrightarrow (+, +)$

$$|z| = r = \sqrt{2^2 + (2\sqrt{3})^2} = 4. \quad 1^{\text{st}} \text{ quad.}$$

$$\tan\theta = \frac{2\sqrt{3}}{2} \Rightarrow \theta = 60 \text{ or } \frac{\pi}{3}$$

$$\therefore Z = 4e^{\frac{\pi}{3}i}. \quad \text{(b)}$$

[14] $\sin^2\theta_x + \sin^2\theta_y + \sin^2\theta_z$

$= 1 - \cos^2\theta_x + 1 - \cos^2\theta_y + 1 - \cos^2\theta_z$

$= 3 - [\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z]$
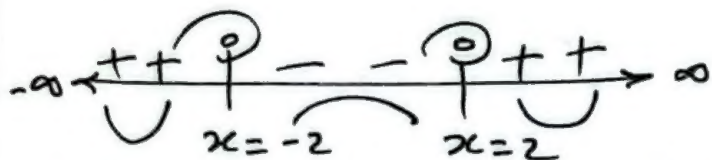
$= 3 - 1 = 2$. ⓒ

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[15] $f(x) = x^4 - 24x^2 + 4$

$f'(x) = 4x^3 - 48x$

$f''(x) = 12x^2 - 48$

<u>Put $f''(x) = 0$</u> : $12x^2 - 48 = 0$

$x^2 = 4 \Rightarrow x = \pm 2$

∴ $f(x)$ is convex down. in interval

$]-\infty, -2[\cup]2, \infty[$

$= \mathbb{R} - [-2, 2]$. ⓓ

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[16] $Z = -\sqrt{3} + i \rightarrow (-, +)$

2nd quad.

$r = |Z| = \sqrt{(-\sqrt{3})^2 + 1^2} = 2$.

$\theta = 180° + \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = 150°$

$Z = 2(\cos 150° + i \sin 150°)$. ⓑ

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[17] $\vec{A} = (1, -2, 5)$ and

$\vec{n} = (2, 1, 3)$

$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$

$(2,1,3) \cdot (x,y,z) = (1,-2,5) \cdot (2,1,3)$

$2x + y + 3z = 15$. ⓑ

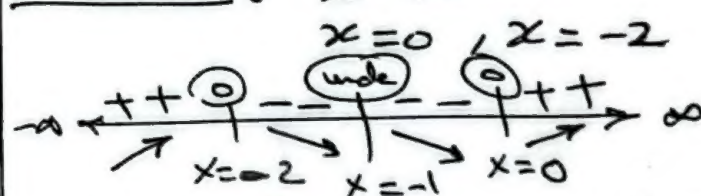[18] $f(x) = \frac{x^2+x+1}{x+1} \Rightarrow$ domain $= \mathbb{R} - \{-1\}$

$f'(x) = \frac{(x+1)(2x+1) - (x^2+x+1)(1)}{(x+1)^2}$

$= \frac{2x^2 + 3x + 1 - x^2 - x - 1}{(x+1)^2}$

∴ $f'(x) = \frac{x^2 + 2x}{(x+1)^2}$

<u>Put $f'(x) = 0$</u> : $x^2 + 2x = 0$

$x = 0, \quad x = -2$

∴ $f(x)$ decreasing in $]-2, 0[- \{-1\}$

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[19] $ab = (2 + 3\omega)(2 + 3\omega^2)$

$= 4 + 6\omega^2 + 6\omega + 9\omega^3$

$= 4 - 6 + 9 = 7$.

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[20] let $x =$ smaller

$y =$ between, and $z =$ greatest

∵ $x + y + z = 36$ & $z = 2x$

∴ $y = 36 - 3x$

Product $= xyz = x(36-3x)(2x)$

$P(x) = 2x^2(36 - 3x) = 72x^2 - 6x^3$

$\frac{dP}{dx} = 144x - 18x^2 \Rightarrow$ Put $\frac{dP}{dx} = 0$:

$\frac{d^2P}{dx^2} = 144 - 36x \qquad ∴ 144x - 18x^2 = 0$

at $x = 8$, then $\frac{d^2P}{dx^2} < 0$ $\qquad x = 0, x = 8$ refused

∴ $P(x)$ has a max. value at $x = 8$.

then the numbers $x = 8, y = 12, z = 16$.